

Homework 3

Due on May 2nd 2018 (at the beginning of class)

NOTE: Before you solve the problems included here, remember to go back to your last problem set and finish all the oligopoly pricing models (Bertrand, Cournot, and Stackelberg) that we have not covered on Midterm 2. I will not ask you to formally submit these as part of this assignment, but you should know them for your final exam.

Problem I: Consider the following situation. There are M sellers of used cellphones on the market and many potential buyers who compete in Bertrand fashion to buy these cellphones. Assume cellphones can either be of good quality or bad quality. The reservation seller prices are \$150 for a good quality phone, and \$20 for a bad quality phone. Buyers are willing to pay up to \$200 for a good quality phone, and up to \$50 for a bad quality phone. The ratio of good phones to bad phones among these M sellers is $1/2$. That means that for every good phone there are two bad phones or that the probability of dealing with a good phone is $1/3$ and the probability of dealing with a bad phone is $2/3$.

- a) What will happen if buyers can determine the exact quality of the phones before buying? What goods get traded, what are the prices, and what is the total market surplus in this full information case? ($\mathbf{TS} = \frac{110}{3}\mathbf{M}$)
- b) What happens under asymmetric information? Just as before, state which goods get traded, at what prices, and what is the total surplus on the market? ($\mathbf{TS} = 20\mathbf{M}$)
- c) What should be the seller's reservation price for the high quality phone (instead of \$150) such that we will not observe an adverse selection problem? (≤ 100)

Problem II: Assume the owner of a large ranch is considering selling his land. There are many potential buyers interested in buying his land who compete a la Bertrand for the right to buy this land. They believe there might be oil under his property. Assume an investor would be willing to pay him \$23 mil. if there was oil for sure and \$5 mil. if there was no oil. Assume the rancher would be willing to sell his land for \$17 mil. if there was oil and for \$3 mil. if there was no oil. Assume there is a 50-50 chance that there is oil indeed under his property.

- a) What is the outcome if neither the rancher, nor the investors know for sure whether there is oil or not? Calculate the expected profits and also explain the profits or losses in each possible state of nature. ($\mathbf{E}(\pi) = 4$)
- b) Assume now that the rancher knows for a fact whether there is oil or not, but the investors still don't know (asymmetric information). What will be the outcome under this scenario?

Problem III: Consider the following problem between a student and his parents. The student doesn't care which grade he gets, as long as he doesn't fail. The student knows that if he doesn't put any effort he will fail. For any passing grade, let the student's utility be $U = 500 - c$, where c is the cost of effort. The parents get a utility of 1000 if he gets an A, 600 if he gets a B, and 400 if he gets a C. Assume that his success in the exam depends on the

level of effort he puts into studying and also on his luck (the professor asks him questions he knows how to solve). Assume the probability of bad and good luck is $1/2$. Also assume that if he works hard he gets either an A or a B (depending on his luck) while if he only puts low effort he gets either a B or a C, again depending on luck. If he doesn't work at all he will fail for sure which he will avoid at all costs. Assume high effort costs him 10 hours of studying, while low effort only costs him 2 hours of studying. Assume his valuation for an hour of free time is \$10.

- a) Assume parents can monitor the student's effort and give him incentives to work hard. Assume they give him money to work hard. What is the prize that they should give him conditional on him working hard? Assume each dollar given to the kid, costs the parents a utility of 1. What is the parents' expected utility? (**$E(U) = 720$**)
- b) Is that profitable for the parents? Should they give him incentives or let him do whatever he wants?
- c) Assume now that parents can no longer monitor the student's effort but they can see his grades. What will happen if they still promise him the same prize as before if he puts high effort? What will the parents' utility be in this case? (**$E(U) = 420$**)
- d) What is the prize for a grade of A that they should promise him in order for him to work hard? What will parents' utility be under this scenario? How about student's expected utility? Does the total surplus change or not? Why? (**prize = 160**)
- e) Assume now that the student's utility is given by $U = \sqrt{500 - c}$. What is the risk premium that parents should pay to the student in order for him to still work hard under the new utility function? is this still profitable for the parents? (**RiskPremium = 7.29**)

Problem IV: An employer is considering to hire an employee. He will offer a contract, after which the employee will decide whether to work hard or shirk. The employer's profit depends on the effort exerted by the employee and also on chance. Assume that if the worker works hard the profit will be \$700 if he gets lucky and \$500 if not. If the employee shirks, the profit will be \$500 if he gets lucky and \$100 if not. Assume the probability of getting lucky is $1/2$. The worker's outside option is \$100 and working hard costs \$100.

- a) What is the optimal contract the employer should offer if effort can be monitored? What is the total surplus in this case? (**TS = 400**)
- b) What if effort is not observable? What is the incentive contract that the employer should offer? Assume the agent is risk neutral. (Note: an incentive contract should be something like this: I pay you \$W no matter what, but if I get \$700 profit then I pay you an additional bonus of \$X.) What is the total surplus? (**TS = 400**)
- c) Explain what would happen if the agent is risk averse. Two or three lines will suffice. Clearly state what needs to happen to the bonus paid and why? How will the agent's utility, principal's profit, and total surplus change as a result of that?

Problem V: There are two people who apply for a job. One is highly skilled, the other one is not. Their productivities (profits that they bring to the employer) are $\Pi_h = 1000$ and $\Pi_l = 300$. Their reservation wages are $w_h = 500$ and $w_l = 100$.

- a) Assume the employer can distinguish who is highly skilled and who is not. Who will get hired and at what wage? Assume he can hire both or just one of them depending on which is more profitable. What will be the profit for the employer? (**$\pi = 700$**)
- b) Assume the employer cannot distinguish between workers. What is the wage that he would

offer, who will get hired and what will the the profit for the firm in this case? ($\pi = 300$)

c) Can the employer do better than this? Design an optimal mechanism based on signaling through education. Assume the costs for one year of education are $c_h = 10$ and $c_l = 50$. You need to find wages for the two types, and a level of education that results in a separating equilibrium (is both individually rational and incentive compatible). What will be the employer's profit in this case? What is the cost of information associated with this problem? ($\pi = 600$)

Problem VI: Apple used to produce two different versions of the Iphone, Iphone 5s and Iphone 5c. Assume the cost of production is zero. There are two types of consumers on the market, knowledgeable consumers and kids. Knowledgeable consumers value the Iphone 5s at \$220 for its superior technology and the Iphone 5c at only \$120. Kids don't know that much about technology and they like colors a lot so their valuations are \$110 for the Iphone 5s and \$100 for the Iphone 5c. Apple is designing a pricing strategy to sell the Iphone.

a) Apple is considering a price discrimination strategy to sell the Iphone 5s to knowledgeable customers and the Iphone 5c to kids for two different prices. Write down the individual rationality and incentive compatibility constraints for the two types of customers. Find the optimal prices. ($P_L = 100, P_H = 200$)

b) Assume there are N total consumers and the proportion of knowledgeable consumers is ϕ . That means there are ϕN knowledgeable consumers and $(1 - \phi)N$ kids on the market. Write the profit function of Apple if the price discrimination mechanism is put into action.

c) Apple is also considering not selling to kids at all and only offering the Iphone 5s for \$220 to knowledgeable consumers. What would be the profit in this case?

d) For which values of ϕ will Apple find it profitable to offer both types of phones and sell to both knowledgeable consumers and kids? ($\phi \leq 5/6$)

Problem VII: A monopolist can produce two slightly different versions (qualities) of a product. There are two types of customers on the market: high types with valuations 250, and 150 respectively for the high and low quality, and low types with valuations 100, respectively 50 for the high and low quality. Assume there is only one consumer of each type. What is the optimal pricing strategy for the monopolist? Price discrimination or just offering a single quality and price? Assume it costs nothing to produce these products and consumers only buy one unit of the good (either high or low quality). ($\pi = 250$)

Problem VIII: A popular clothing company sells t-shirts on a market populated by two types of consumers: snobs and commoners. There are 100 snobs and 200 commoners on this market, but it is impossible for the firm to distinguish snobs from commoners. The company can produce two types of t-shirts: branded t-shirts with a large company logo conspicuously placed on the chest, and non-branded t-shirts with only a very small logo on the sleeve. Snobs place a valuation of \$200 for a branded t-shirt and a valuation of \$100 for a non-branded t-shirt. Commoners valuations are \$80 for a branded t-shirt, respectively \$50 for a non-branded one. Assume for simplicity that the costs of production are zero.

a) Assuming a uniform pricing strategy (only one type of t-shirt produced and sold at a uniform price), what is the maximum profit that the firm can earn? ($\pi = 24000$)

b) Assuming a second degree price discrimination strategy (both t-shirts are sold for different prices to different customers), what are the prices and the profit for the firm? ($\pi = 25000$)

c) What is the most profitable strategy if we now have 250 snobs and 200 commoners on this market? Specify the price/prices and the resulting profit. Don't guess, analyze both strategies (uniform pricing and price discrimination) and choose the one with higher profits. ($\pi = \mathbf{50000}$)

d) Suppose we realize all snobs have a tattoo on their forehead or something similar that makes them easily identifiable and suppose we can prevent resales between commoners and snobs. How should we price these products and what would be the resulting profit? Do this for both of the above scenarios (100/200 and 250/200 snobs to commoners) and compare to your best profits from before. ($\pi = \mathbf{36000}/\pi = \mathbf{66000}$)

Problem IX: Consider the following valuations table for five consumers willing to purchase two differently colored T-shirts:

Consumer	Valuation for Blue T-shirts	Valuation for Pink T-shirts
John	50	15
Bob	40	30
Tim	35	35
Jennifer	25	45
Maria	10	50

a) Assuming no production costs, what is the most profitable way to price these two goods independently? What are the prices and resulting profit? ($\pi = \mathbf{225}$)

b) Assuming no production costs, what is the most profitable way to price these two goods using bundling? Analyze both pure and mixed bundling, state the prices and resulting profits and choose the best option. ($\pi = \mathbf{310}$)

c) Assuming costs of production of \$27 per t-shirt (regardless of color), what is the most profitable bundling pricing strategy? Specify whether it's better to use mixed or pure bundling, then record the optimal prices and resulting profit. ($\pi = \mathbf{94}$)

Problem X: The market demand for a product is given by $P = 30 - 3Q$. Assume the marginal cost of production is constant, $MC = 6$.

a) What is the optimal two part tariff for a producer on this market? What is the resulting profit?

b) Assume now that there are two groups of consumers, one with the demand given above, and another group with a higher demand given by $P = 30 - 2Q$. Also assume marginal cost is zero. What is the optimal two part tariff and the resulting profit under this scenario?